APPLICATION OF THE LAPLACE TRANSFORM TO CALCULATE THE VELOCITY OF A TWO-PHASE FLUID MODULATED BY THE MOVEMENT OF CUTTINGS OF AN ENERGY WILLOW (SALIX VIMINALIS)

Serhii Yermakov

Educational and scientific laboratory "DAK GPS", State Agrarian and Engineering University
32-300 Kamianets-Podilskyi, Ukraine.
e-mail: ermkov@gmail.com

Abstract. In order to create automatic systems for planting energy willow, it is important to study the process of gravitational unloading of cuttings. When constructing a mathematical model of this process, planting material was characterized as a pseudo-fluid consisting of a discrete component (cuttings) and a gaseous medium (air), and the gravity unloading itself was considered from the point of view of hydrodynamic multiphase systems using the corresponding general equations for characterizing the motion. By applying the Laplace transform to determine the Fourier coefficients, we obtain a system of linear algebraic equations of the velocity of motion of such a pseudo-fluid.

Keywords: energy willow (salix viminalis), planting automation, mathematical model, Laplace transform, multiphase system

INTRODUCTION

As theoretical studies, in the direction of unloading bulk materials, show, this problem is far from solved. Numerous studies of the process of gravitational outflow of materials made it possible to establish only some dependencies that explain the essence of this process (Bagnold, 1954; Bohomiahkih, Pepchuk, 1985; Gyachev, 1992; Savage, Cowin, 1999; Zenkov *et al.* 1966). It is due to the complexity of ensuring uniform continuous movement till now, there is no universal power device that works effectively with any bulk material, and the variety of material requiring unloading contributes to a further search for justifications for the movement of a particular material. So in this study, such material is cuttings of plants. The need to study this issue is dictated by the increasing popularity of fuels from bioenergy crops, which require fast and efficient machines to create the so-called energy plantations to increase their volumes. The most common energy willow in Ukraine is vegetatively propagated by cuttings 20-25 cm long and 5-20 mm in diameter (Frączek, Mudryk, 2005; Dziedzic *et al.* 2017; Hutsol *et al.* 2018; Yermakov *et al.* 2018).

Today, such material is planted with planters in which planting material is supplied exclusively by hand, which significantly limits the possibility of increasing the efficiency of the units. A theoretical study of the cuttings' movement and the implementation of the results in practice can be of help in creating a planting machine.

Figure 1. Planting material of energy willow

In accordance with the scientific direction at the State Agrarian and Engineering University in Podilia “Research on workflow and parameters of the feeling mechanism for cuttings in energy willow planting” (state registration number 0119U100945), an automated system for feeding and selecting planting material for wood energy crops is being developed.

One of the first steps in this work is the construction of a mathematical model of the process of gravitational flow of rod-shaped materials from slot hoppers (Yermakov, Hutsol 2018; Yermakov *et al.* 2019).

MATERIALS AND METHODS

In previous works, we have already worked out general principles for constructing a mathematical model of the process of unloading cuttings from the hopper, determined the boundary conditions and characteristics of their movement (Yermakov *et al.* 2018; Yermakov, Hutsol, 2018; Yermakov *et al.* 2019).

The model of the hopper was taken as the basis (Fig. 2), in which the consideration of the process is limited to the two-dimensional model (in the plane $x_1 x_2$), since it is assumed that the movement of cuttings in the hopper is independent of the coordinate x_3, due to the presence of walls parallel to the plane $x_1 x_2$ that restrict movement cuttings along the axis x_3.

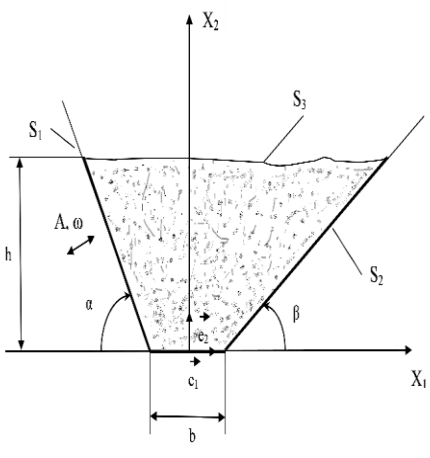

Figure 2. The design scheme of the hopper with cuttings

Moreover, based on an analysis of existing solutions, a number of assumptions were made, which made it possible to consider the gravitational unloading of cuttings from the point of view of hydrodynamic multiphase systems. In accordance with this approach, the set of cuttings is considered as a pseudo-fluid consisting of two phases: a discrete, formed by cuttings and a continuous phase (gas-like medium between the cuttings) (Yermakov *et al.* 2019). Each of these phases is considered as a continuous medium, which allowed us to consider unloading as the motion of a viscous incompressible pseudo-fluid, the equations of motion of which were presented in the following form (Sous 1971; Nyhmatulin 1978).

$$\rho\left(\frac{\partial \vec{V}}{\partial t}+\left(\vec{V}_1,\nabla\right)\vec{V}\right)=-\nabla P+\mu\Delta\vec{V}+\vec{F}-\rho g e_2\,, \tag{1}$$

$$\rho_1\left(\frac{\partial \vec{V}_1}{\partial t}+\left(\vec{V}_1,\nabla\right)\vec{V}_1\right)=-(1-\delta)\nabla P_1+\mu_1\Delta\vec{V}_1-\vec{F}-\rho_1 g e_2\,, \tag{2}$$

$$div\vec{V}=0\,, \tag{3}$$

$$div\vec{V}_1=0\,, \tag{4}$$

where e_1, e_2, e_3 are the unit vectors of the Cartesian coordinate system,

$\vec{V}, \vec{V}_1$ - velocity field of discrete and continuous phases,

P, P_1 - pressure of discrete and continuous phases,

g - acceleration of gravity.

RESULTS AND DISCUSSION

Basic assumptions about the nature of the motion of a two-phase pseudo-fluid. In the previous works of the authors (Yermakov *et al.* 2019), the formulation of the initial-boundary-value problem of the motion of a two-phase pseudo-fluid is given, which simulates the process of unloading cuttings from the hopper. The equation of motion of this pseudo-fluid is nonlinear integrodifferential equations. This significantly complicates the solution to these equations. However, under certain assumptions about the nature of the motion of the pseudo-fluid, these equations can be simplified. This made it possible to obtain their solutions in an analytical form.

Let us formulate these assumptions. To do this, we introduce dimensionless variables by the formulas

$$t = T\cdot\bar{t},\, x_1 = L\cdot\bar{x}_1, x_2 = L\cdot\bar{x}_2, \quad \vec{V}_1 = V_0\cdot\vec{U}_1, \vec{V} = V_0\cdot\vec{U} \tag{5}$$

where T is the characteristic time, L is the characteristic length size, V_0 is the maximum velocity of the pseudo-fluid.

In these variables, the equation of motion (1) - (4) will take the form

$$\frac{\partial\vec{u}}{\partial\bar{t}} + \frac{V_0T}{L}(\vec{u},\nabla)\vec{u} = -\frac{T}{V_0\rho L}\nabla\rho + \frac{\nu T}{L^2}\Delta\vec{u} + 0.5\frac{\bar{\rho}_1}{\bar{\rho}}\left[\frac{\partial}{\partial\bar{t}}(\vec{u}_1-\vec{u}) + \frac{V_0T}{L}\times(\vec{u}_1-\vec{u},\nabla)(\vec{u}_1-\vec{u})\right] + \frac{4.5\bar{\rho}_1\sqrt{\nu_1T}}{\sqrt{\pi}a\bar{\rho}}\int_0^{\bar{t}}\left(\frac{\partial}{\partial\bar{\tau}}(\vec{u}_1-\vec{u}) + \frac{V_0T}{L}\times(\vec{u}_1-\vec{u},\nabla)(\vec{u}_1-\vec{u})\right)\times(\bar{t}-\bar{\tau})^{-1/2}d\bar{\tau} + 3.75\frac{\bar{\rho}_1\delta\nu_1T}{\bar{\rho}a^2(1-\delta)^2}(\vec{u}_1-\vec{u}) - g\frac{T}{V_0}\vec{e}_2. \tag{6}$$

$$\frac{\partial\vec{u}_1}{\partial\bar{t}} + \frac{V_0T}{L}(\vec{u}_1,\nabla)\vec{u}_1 = -\frac{(1-\delta)T}{V_0\rho_1L}\nabla\rho_1 + \frac{\nu_1T}{L^2}\Delta\vec{u}_1 - 0.5\frac{\delta}{1-\delta}\left[\frac{\partial}{\partial\bar{t}}(\vec{u}_1-\vec{u}) + \frac{V_0T}{L}(\vec{u}_1-\vec{u},\nabla)(\vec{u}_1-\vec{u})\right] - \frac{4.5\delta\sqrt{\nu_1T}}{\sqrt{\pi}a(1-\delta)}\int_0^{\bar{t}}\left(\frac{\partial}{\partial\bar{\tau}}(\vec{u}_1-\vec{u}) + \frac{V_0T}{L}(\vec{u}_1-\vec{u},\nabla)(\vec{u}_1-\vec{u})\right)\times(\bar{t}-\bar{\tau})^{-1/2}d\bar{\tau} - 3.75\frac{\delta^2\nu_1T}{a^2(1-\delta)^3}(\vec{u}_1-\vec{u}) - g\frac{T}{V_0}\vec{e}_2. \tag{7}$$

$$\operatorname{div}\vec{u} = 0, \quad \operatorname{div}\vec{u}_1 = 0. \tag{8}$$

From (6), (7) it follows that if the quantity V_0T/L is sufficiently small

$$\frac{V_0T}{L} << 1, \tag{9}$$

then in equations (6), (7) we can neglect nonlinear members of the type $(\vec{u},\nabla)\vec{u}$, $(\vec{u}_1-\vec{u},\nabla)(\vec{u}_1-\vec{u})$. As the quantities V_0, T, L we choose the following; $V_0 = \omega A$ - maximum speed of vibrations, $T = 2\pi/\omega$ - period of vibrations, $L = 2\pi a$ - circumference of the cross-section of the cutting. Then inequality (9) takes the form

$$\frac{A}{a} << 1. \tag{10}$$

Thus, if the amplitude of the vibrations is sufficiently small, then in equations (6), (7) the nonlinear terms are small. In what follows, we will assume that inequality (10) holds.

Further, since $\vec{\rho}_1$ is the density of air, $\bar{\rho}$ is the averaged density of cuttings, the quantity is

$$\frac{\bar{\rho}_1}{\bar{\rho}} << 1. \tag{11}$$

Therefore, the third and fourth terms on the right side of equation (6) can be neglected.

In addition, since the volume concentration of the cuttings $\delta \approx 1$, and the coefficient of kinematic viscosity of the air (continuous phase) ν_1 is small ($\nu_1 \approx 10^{-5}\, m^2/s$), the first and second terms on the right side of equation (7) can be neglected.

Thus, based on the assumptions made about the nature of the motion of the two-phase pseudo-fluid, equations (6) - (8) are simplified and take the form

$$\frac{\partial\vec{u}}{\partial\bar{t}} = -\gamma\nabla\rho + \bar{\nu}\Delta\vec{u} + \Phi(\vec{u}_1-\vec{u}) - \bar{g}\vec{e}_2, \tag{12}$$

$$\frac{\partial\vec{u}_1}{\partial\bar{t}} = -0.5\frac{\delta}{1-\delta}\frac{\partial}{\partial\bar{t}}(\vec{u}_1-\vec{u}) - \Phi_1\int_0^{\bar{t}}\frac{\partial}{\partial\bar{t}}(\vec{u}_1-\vec{u})(\bar{t}-\bar{\tau})^{-1/2}d\bar{\tau} - \Phi_2(\vec{u}_1-\vec{u}) - \bar{g}\vec{e}_2, \tag{13}$$

$$\operatorname{div}\vec{u}=0,\quad \operatorname{div}\vec{u}_1=0. \tag{14}$$

The notations are introduced here.

$$\gamma=\frac{T}{V_0\rho L},\quad \bar{g}=g\frac{T}{V_0},\quad \bar{\nu}=\frac{\nu T}{L^2}, \tag{15}$$

$$\Phi=3.75\frac{\bar{\rho}_1\delta\nu_1 T}{\bar{\rho}a^2(1-\delta)^2}, \tag{16}$$

$$\Phi_1=4.5\frac{\delta\sqrt{\nu_1 T}}{\sqrt{\pi}a(1-\delta)}, \tag{17}$$

$$\Phi_2=3.75\frac{\delta^2\nu_1 T}{a^2(1-\delta)^3}, \tag{18}$$

Equations (12) - (14) are the basis for describing the process of unloading cuttings from the hopper. To these equations it is necessary to add initial and boundary conditions, which in the new notation have the form:

Initial conditions

$$\vec{u}\big|_{\bar{t}=0}=\vec{u}_1\big|_{\bar{t}=0}=0,\qquad \rho\big|_{\bar{t}=0}=0, \tag{19}$$

Boundary conditions

if $\bar{x}_2=h(\bar{t})/L$

$$-\rho+\frac{2\mu V_0}{L}\frac{\partial u_2}{\partial\bar{x}_2}=0, \tag{20}$$

$$\frac{\partial u_1}{\partial\bar{x}_2}+\frac{\partial u_2}{\partial\bar{x}_{12}}=0, \tag{21}$$

$$\dot{h}=V_0Tu_2, \tag{22}$$

if $\bar{x}_2=-\operatorname{tg}\alpha\left(\bar{x}_1+b/2L\right),\ /\ -\operatorname{ctg}\alpha h_0/L-b/2L<\bar{x}_1<b/2L$

$$\sin\alpha u_1+\cos\alpha u_2=\frac{A\omega}{V_0}\sin 2\pi\bar{t}, \tag{23}$$

$$\cos 2\alpha\left(\frac{\partial u_1}{\partial\bar{x}_1}+\frac{\partial u_2}{\partial\bar{x}_2}\right)+ +2\sin 2\alpha\frac{\partial u_1}{\partial\bar{x}_1}= =\frac{gL\cos^2\alpha\quad h(\bar{t})}{2\nu V_0}, \tag{24}$$

if

$$\bar{x}_2=\operatorname{tg}\beta(\bar{x}_1-b/2L),\quad b/2L<\bar{x}_1<b/2L+\frac{\operatorname{ctg}\beta h_0}{L}$$

$$\cos 2\beta\left(\frac{\partial u_1}{\partial\bar{x}_2}+\frac{\partial u_2}{\partial\bar{x}_1}\right)-2\sin 2\beta\frac{\partial u_1}{\partial\bar{x}_1}= =\frac{gL\cos^2\beta\quad h(\bar{t})}{2\nu V_0}. \tag{25}$$

*Application of the Laplace transform to the equations of motion of a two-phase pseudo-fluid.*The solutions of equations (12) - (14) can be found using the Laplace transform.

Before applying the Laplace transform, we determine the pressure ρ. To do this, we act by differential operation div on the left and right sides of equation (12). Then, taking into account equations (14), we have

$$\Delta\rho=0. \tag{26}$$

Assuming that the pressure ρ is weakly dependent on the variable x_1

$$\left|\frac{\partial^2\rho}{\partial\bar{x}_1{}^2}\right|<<\left|\frac{\partial^2\rho}{\partial\bar{x}_2{}^2}\right|$$

from (26) we obtain

$$\left|\frac{\partial^2\rho}{\partial\bar{x}_2{}^2}\right|=0. \tag{27}$$

Integrating (27), we have

$$\rho=C_1\bar{x}_2+C_2, \tag{28}$$

where $C_1$and C_2 are constant values independent of the variable $\bar{x}_2$. The constants

$C_1$and C_2 can be determined from the boundary conditions on the free surface of the layer of cuttings and on the border of the discharge window of the hopper.

Neglecting the influence of the atmosphere and assuming that at the boundary of the discharge window the pressure coincides with the standard pressure $p_{cm} = \rho g h$, we obtain

$$p = \rho g\left(h - L\bar{x}_2\right). \tag{29}$$

Let us substitute (29) into (12). After a series of transformations, we have

$$\frac{\partial \vec{u}}{\partial \bar{t}} = \bar{\nu}\Delta\vec{u} + \Phi\left(\vec{u}_1 - \vec{u}\right). \tag{30}$$

We introduce the Laplace transform with respect to the time variable $\bar{t}$ for the velocity fields $\vec{u}$ and $\vec{u}_1$ of the discrete and continuous phases of the pseudo-fluid

$$\vec{U} = \int_0^\infty \vec{u}e^{-q\bar{t}}d\bar{t}, \quad \vec{U}_1 = \int_0^\infty \vec{u}_1 e^{-q\bar{t}}d\bar{t}. \tag{31}$$

Let us apply the Laplace transform to (30) and (13), (14). Then, taking into account (31), using the property of the Laplace transform and the formula

$$\int_0^\infty e^{-q\bar{t}}\int_0^{\bar{t}} U(\tau)\left(\bar{t}-\tau\right)^{-1/2}d\tau = \sqrt{\pi q}\int_0^\infty U(\bar{t})e^{-q\bar{t}}d\bar{t}$$

we have

$$q\vec{U} = \bar{\nu}\Delta\vec{U} + \Phi\left(\vec{U}_1 - \vec{U}\right), \tag{32}$$

$$\vec{U}_1\Psi = \left(\Psi - q\right)\vec{U} - \frac{\vec{\bar{g}}}{q}\vec{e}_2, \tag{33}$$

$$\operatorname{div}\vec{U} = 0, \tag{34}$$

where

$$\Psi = q\frac{1-0.5\delta}{1-\delta} + \sqrt{\pi q}\Phi_1 + \Phi_2. \tag{35}$$

Excluding $\vec{U}_1$ from (32), (33) we have

$$\Delta\vec{U} = \frac{q}{\bar{\nu}}\left(1 + \frac{\Phi}{\Psi}\right)\vec{U} + \frac{\Phi\bar{g}}{q\bar{\nu}\Psi}\vec{e}_2, \tag{36}$$

$$\operatorname{div}\vec{U} = 0. \tag{37}$$

Having determined from (36), (37) the Laplace transform $\vec{U}_1$ of the discrete phase velocity, from (33) it is possible to determine the Laplace transform of the continuous phase velocity

$$\vec{U}_1 = \left(1 - \frac{q}{\Psi}\right)\vec{U} - \frac{\bar{g}}{q\Psi}\vec{e}_2. \tag{38}$$

Thus, to solve the problem, it is sufficient to determine the velocity of the discrete phase of the pseudo-fluid.

In addition to equations (36), (37), it is necessary to add boundary conditions (23) - (25), which, after applying the Laplace transform, are reduced to

if $\bar{\bar{x}}_2 = -\operatorname{tg}\alpha\left(\bar{x}_1 + b/2L\right)$

$$\sin U_1 + \cos U_2 = \frac{A\omega 2\pi}{V_0\left(q^2 + 4\pi^2\right)}, \tag{39}$$

$$\cos 2\alpha\left(\frac{\partial U_1}{\partial \bar{x}_1} + \frac{\partial U_2}{\partial \bar{x}_2}\right) + 2\sin 2\alpha\frac{\partial U_1}{\partial \bar{x}_1} = \frac{fgL\cos^2\alpha\bar{h}}{2\nu V_0}, \tag{40}$$

if $\bar{\bar{x}}_2 = -\operatorname{tg}\beta\left(\bar{x} - b/2L\right)$

$$\cos 2\beta\left(\frac{\partial U_1}{\partial \bar{x}_2} + \frac{\partial U_2}{\partial \bar{x}_1}\right) - 2\sin 2\beta\frac{\partial U_1}{\partial \bar{x}_1} = \frac{fgL\cos^2\beta\bar{h}}{2\nu V_0}, \tag{41}$$

where

$$\bar{h} = \int_0^\infty h(\bar{t})e^{-q\bar{t}}d\bar{t}. \tag{42}$$

Let us find a solution to problem (36), (37). To do this, we pass from the vector form to the scalar one.

$$\Delta U_1 - \lambda U_1 = 0, \tag{43}$$

$$\Delta U_2 - \lambda U_2 = d, \tag{44}$$

$$\frac{\partial U_1}{\partial \bar{x}_1} + \frac{\partial U_2}{\partial \bar{x}_2} = 0, \tag{45}$$

where

$$\vec{U} = U_1\vec{e}_1 + U_2\vec{e}_2, \tag{46}$$

$$\lambda = \frac{q}{\bar{\nu}}\left(1+\frac{\Phi}{\Psi}\right), \tag{47}$$

$$d = \frac{\Phi\bar{g}}{q\bar{\nu}\Psi}. \tag{48}$$

Equations (43) - (45) must be satisfied in the volume of the hopper. Continue the functions $U_1$and U_2 beyond the hopper to the rectangle

$$-\frac{h_0}{L}\operatorname{ctg}\alpha - \frac{b}{2L} \le \bar{x}_1 \le \frac{h_0}{L}\operatorname{ctg}\beta + \frac{b}{2L}, \tag{49}$$

$$0 \le \bar{x}_2 \le \frac{h_0}{L}$$

In the following way: U_1 will continue with zero, and U_2 with a constant $-d/\lambda$.

Then equations (43) - (45) will be satisfied on the entire rectangle (49).

We will seek a solution to these equations in the form of Fourier series in the variable $\bar{x}_1$

$$U_1 = \sum_{n=0}^{\infty} A_{1n}\cos\frac{2\pi}{M}n\bar{x}_1 + A_{2n}\sin\frac{2\pi}{M}n\bar{x}_1, \tag{50}$$

$$U_2 = -\frac{d}{\lambda} + \sum_{n=0}^{\infty} B_{1n}\cos\frac{2\pi}{M}n\bar{x}_1 + \\ + B_{2n}\sin\frac{2\pi}{M}n\bar{x}_1, \tag{51}$$

where $M = b/L + (\operatorname{ctg}\alpha + \operatorname{ctg}\beta)h_0/L$.

Here the quantities $A_{1n}, A_{2n}, B_{1n}, B_{2n}$ are unknown functions of the variable $\bar{x}_2$.

To find these functions, we substitute (50) and (51) into equations (43) and (44). Making the necessary calculations, we will have

$$\ddot{A}_{1n} - \lambda_n A_{1n} = 0, \quad \ddot{A}_{2n} - \lambda_n A_{2n} = 0, \tag{52}$$

$$\ddot{B}_{1n} - \lambda_n B_{1n} = 0, \quad \ddot{B}_{2n} - \lambda_n B_{2n} = 0, \quad n = 0,1,... \tag{53}$$

where

$$\lambda_n = \lambda + \left(\frac{2\pi n}{M}\right)^2, \tag{54}$$

and the dot denotes the operation of differentiation with respect to the variable $\bar{x}_2$.

Next, we substitute (50) and (51) into equation (45). We will obtain

$$\dot{B}_{1n} + \frac{2\pi n}{M}A_{2n} = 0, \quad \dot{B}_{2n} - \frac{2\pi n}{M}A_{1n} = 0. \tag{55}$$

The general solution of equations (52), (53) can be represented as

$$A_{1n} = \bar{A}_{1n}e^{-\sqrt{\lambda_n}\bar{x}_2} + C_{1n}e^{\sqrt{\lambda_n}\bar{x}_2},$$
$$A_{2n} = \bar{A}_{2n}e^{-\sqrt{\lambda_n}\bar{x}_2} + C_{2n}e^{\sqrt{\lambda_n}\bar{x}_2},$$
$$B_{1n} = \bar{B}_{1n}e^{-\sqrt{\lambda_n}\bar{x}_2} + D_{1n}e^{\sqrt{\lambda_n}\bar{x}_2},$$
$$B_{2n} = \bar{B}_{2n}e^{-\sqrt{\lambda_n}\bar{x}_2} + D_{2n}e^{\sqrt{\lambda_n}\bar{x}_2}, \tag{56}$$

where $\bar{A}_{1n}, \bar{A}_{2n}, C_{1n}, C_{2n}, \bar{B}_{1n}, B_{2n}, D_{1n}, D_{2n}$ are arbitrary constants.

Since λ_n depends on the parameter q of the Laplace transform (see (31)), which, generally speaking, is a complex number, for the root $\sqrt{\lambda_n}$ we choose the branch for which $\operatorname{Re}\sqrt{\lambda_n} \ge 0.$

As follows from (56), the sought quantities are the sum of two terms. One of them with the increase $n \to \infty$ increases exponentially, while the other decreases. For the convergence of infinite series (50) and (51), exponentially growing terms should be discarded. Therefore, exponentially decreasing terms should be left in (56).

$$A_{1n} = \bar{A}_{1n}e^{-\sqrt{\lambda_n}\bar{x}_2},$$
$$A_{2n} = \bar{A}_{2n}e^{-\sqrt{\lambda_n}\bar{x}_2},$$
$$B_{1n} = \bar{B}_{1n}e^{-\sqrt{\lambda_n}\bar{x}_2},$$
$$B_{2n} = \bar{B}_{2n}e^{-\sqrt{\lambda_n}\bar{x}_2}, \tag{57}$$

Now we substitute (56) into (57). Then we have

$$\bar{A}_{1n} = -\sqrt{\lambda_n}\frac{M}{2\pi n}\bar{B}_{2n},$$
$$\bar{A}_{2n} = \sqrt{\lambda_n}\frac{M}{2\pi n}\bar{B}_{1n}. \tag{58}$$

In particular, it follows from (58) that $\overline{B}_{10} = \overline{B}_{20} = 0$.

In view of the above, for solving equations (43) - (45), we obtain the following formulas

$$U_1 = \overline{A}_{10} e^{-\sqrt{\lambda}\overline{x}_2} + \sum_{n=1}^{\infty} \overline{\lambda}_n e^{-\sqrt{\overline{\lambda}_n}\overline{x}_2} \times \left(\overline{B}_{1n} \sin \frac{2\pi n}{M} \overline{x}_1 - \overline{B}_{2n} \cos \frac{2\pi n}{M} \overline{x}_1 \right), \quad (59)$$

$$U_2 = -d/\lambda + \sum_{n=0}^{\infty} e^{-\sqrt{\overline{\lambda}_n}\overline{x}_2} \times \left(\overline{B}_{1n} \cos \frac{2\pi n}{M} \overline{x}_1 + \overline{B}_{2n} \sin \frac{2\pi n}{M} \overline{x}_1 \right), \quad (60)$$

where

$$\overline{\lambda}_n = \sqrt{\lambda \left(\frac{M}{2\pi n} \right)^2 + 1}. \quad (61)$$

Formulas (59), (60) give a general solution to the system of equations (43) - (45).

CONCLUSIONS

1. The construction of a mathematical model of the motion of cuttings of energy willow will automate the planting process. To date, planters are known exclusively with manual laying of planting material.

2. The easiest way to move material during unloading is to move it under the action of gravitational forces. The theoretical justifications for such a movement do not have a single approach, and the specifics of the material for planting energy willow create additional difficulties for the development of a mathematical model of this process.

3. Taking a number of assumptions, it is proposed to consider the gravitational unloading of cuttings from the point of view of hydrodynamic multiphase systems. In accordance with this approach, the set of cuttings is considered as an incompressible pseudo-fluid consisting of two phases: a discrete, formed by cuttings and a continuous phase (gas-like medium between the cuttings).

4. The basic assumptions about the nature of the motion of a two-phase pseudo-fluid were considered and substantiated, due to which some of the composite equations could be neglected, and the existing equations of motion were significantly simplified.

5. By applying the Laplace transform to determine the Fourier coefficients, a system of linear algebraic equations of the pseudo-fluid velocity has been obtained (see (59), (60)).

REFERENCES

Bagnold R. A. 1954. Experiments on a gravity Free Dispersion of large Solid Spheres in a Newtonian Fluid under Shear // Proc. Roy. Soc. London. Vol. 225: 49 – 63.

Bohomiahkih V. A., Pepchuk A. P. 1985. The improvement of unloading of hopper devices in terms of grain bridging]. Zernohrad (in Russian).

Dziedzic K., Lapczyńska-Kordon B., Mudryk K. 2017 Decision support systems to establish plantations of energy crops on the example of willow (Salix Viminalis L.). Scientific achievements in agricultural engineering, agronomy and veterinary medicine polish Ukrainian cooperation, v. 1, n. 1:. 150-160.

Frączek J., Mudryk K., 2005. Jakości sadzonek wierzby energetycznej w aspekcie sadzenia mechanicznego. Inżynieria Rolnicza, 6 (66): 159-167 (in Polish).

Gyachev L. V. 1992. Fundamentals of hopper theory. Novosibirsk: izd-vo novosibirskogo universiteta. 184p. (in Russian).

Hutsol T., Yermakov S., Firman Ju., Duganets V., Bodnar A. 2018. Analysis of technical solutions of planting machines, which can be used in planting energy willow. Renewable Energy Sources: Engineering, Technology, Innovation. ICORES 2018

Jansen H. 1985. Versuche uber Getreidedruck in Silozellen.-Berlin: S 1045-1049.

Jenkins J. T., Cowin S. C. 1979. Theories for Flowing granular Materials // Mech. Fluid Engng. and Bioengng. Conf., AMD. V. 51: 79 – 89.

Nyhmatulin R. I. 1978. The basics of mechanics of heterogeneous mediums. Moscow: Nauka. 336 p. (in Russian).

Savage S. B., Cowin S. C. 1999. Theories for Flow Granular Materials // American Society of Mechanical Engineers, Buffalo, N.Y., June: 79 – 82

Sous S. 1971. Gidrodinamika mnogofaznyx sistem [Hydrodynamics of multiphase systems]. Moscow: Mir. 536p. (in Russian).

Yermakov S. V. 2017. Perspectives of improvement of constructions for energy crop planting. Podilian Bulletin: agriculture, engineering, economics, v. 2, n. 26: 37-45. (in Ukrainian).

Yermakov S., Hutsol T., Slobodian S., Komarnitskyi S., Tysh M. 2018. Possibility of using automation tools for planting of the energy willow cuttings. Renewable Energy Sources: Engineering, Technology, Innovation. ICORES 2018: 419-429

Yermakov S., Hutsol T. 2018. Features of the heterogeneous rod-like materials outflow. Technological and methodological aspects of agri-food engineering in young scientist research. Krakow: 55-68

Yermakov S., Hutsol, T. Ovcharuk O., Kolosiuk I. 2019. Mathematic simulation of cutting unloading from the bunker. Independent journal of management & production (IJM&P): 758-777

Yermakov S., Mudryk K., Hutsol T., Dziedzic K., Mykhailova L. 2019. The analysis of stochastic processes in unloadingthe energywillow cuttings from the hopper. Environment. Technology. Resources. Rezekne, Latvia. Proceedings of the 12th International Scientific and Practical Conference. Volume III: 249-252

Zenkov R. L., Grinevich G. P., Isaev V. S. 1966. Hopper devices. Moskow: Mashinostroenie. 224p. (in Russian).